IMO Logic, Combinatorics and Spatial Reasoning Questions

Level: Intermediate Ref No: M17 Puzz Points: 23

[Maclaurin 2004 Q5] The magician Mij has 140 green balls and 140 red balls. To perform a trick, Mij places all the balls in two bags. In the black bag there are twice as many green balls as red balls. In the black bag there are twice as many green balls as red balls. In the white bag the number of red balls is a multiple of the number of green balls. Neither bag is empty. Determine all the ways in which Mij can place the balls in the two bags in order to perform the trick.

Solution: (black bag red, black bag green, white bag red, white bag green) $=$ $(56,112,84,28),(60,120,80,20),(68,136,72,4)$

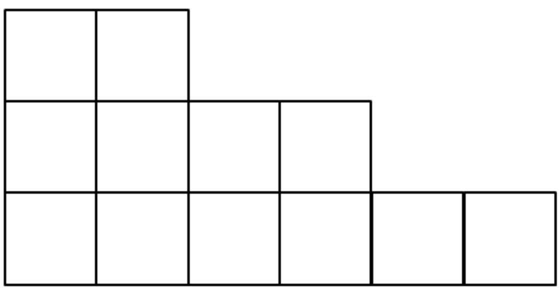
Level: Intermediate Ref No: M36 Puzz Points: 23

[Maclaurin 2006 Q6] I have a large supply of 1p, 2p and 3p stamps.
(a) Explain why there are at least as many ways to make up (*N*+1)p as there are to make up *N*p, for any positive integer *N*.
(b) Explain why the number of ways to make up (*N*+1)p is always greater than the number of ways to make up *N*p.
[We consider 2p, 1p, 1p to be the same way of making up 4p as 1p, 2p, 1p.]

Solution: (a) If q ways of making N p, simply adding a 1p stamp to each of these ways will result in a way of making $(N+1)$ p. Hence at least q ways of making this amount. (b) You need to show there is at least one way of making $(N+1)$p that does not require the use of a 1p stamp. Use case analysis on $N+1$ being even and being odd.

Level: Intermediate Ref No: M37 Puzz Points: 10

[Cayley 2007 Q1] Four copies of the polygon shown are fitted together (without gaps or overlaps) to form a rectangle. How many different rectangles are there?



Solution: Four copies requires 48 squares and 48 can only be factorised as $1\times 48, 2\times 24, 3\times 16, 4\times 12, 6\times 8$. Only three of these are valid rectangles (you need to draw out these three).

Level: Intermediate Ref No: M51 Puzz Points: 20

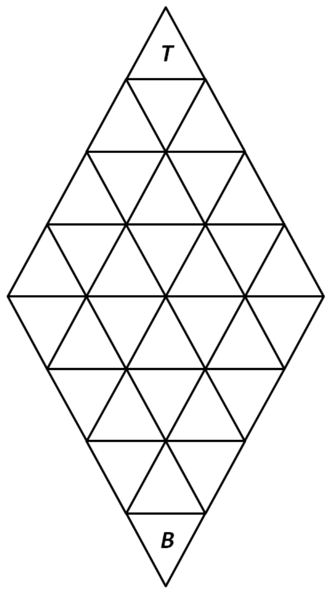
[Maclaurin 2007 Q3] For Erewhon's rail network, it is possible to buy only single tickets from any station on the network to any other station on the network. Each ticket shows the name of the station at which a journey starts and, below this, the name of the destination station.

It is proposed to add several new stations to the network. For how many different combinations of the number of existing stations and the number of new stations will exactly 200 new types of ticket be required?

Solution: 3 (could be 100, 18 or 9 existing stations)

Level: Intermediate Ref No: M60 Puzz Points: 13

1. [Cayley 2011 Q6] A bug starts in the small triangle *T* at the top of the diagram. She is allowed to eat through a neighbouring edge to get to a neighbouring small triangle. So at first there is only one possible move (downwards), and only one way to reach this new triangle.
   (a) How many triangles, including *T* and *B*, must the bug visit if she is to reach the small triangle *B* at the bottom using a route that is as short as possible?
   (b) How many different ways are there for the bug to reach *B* from *T* by a route of this shortest possible length?



Solution: (a) 14 (b) 20

Level: Intermediate Ref No: M65 Puzz Points: 18

[Hamilton 2011 Q5] In how many distinct ways can a cubical die be numbered from 1 to 6 so that consecutive numbers are on adjacent faces? Numberings that are obtained from each other by rotation or reflection are considered indistinguishable.

Solution: 5 ways

Level: Intermediate Ref No: M89 Puzz Points: 23

[Maclaurin 2008 Q6] An artist is preparing to draw on a sheet of A4 paper (a rectangle with sides in the ratio $1{:}\sqrt{2}$). The artist wishes to place a rectangular grid of squares in the centre of the paper, leaving a margin of equal width on all four sides.

Show that such an arrangement is possible for a 1×2 or a 2×3 grid but impossible for any other $g \times (g+1)$ grid.

Solution: Possible for grids of size 1×2 or 2×3 but impossible for any other sizes.

Level: Intermediate Ref No: M117 Puzz Points: 18

[Hamilton 2010 Q6] In the diagram, the number in each cell shows the number of shaded cells with which it shares an edge or a corner. The total of all the numbers for this shading pattern is 16. Any shading pattern obtained by rotating or reflecting this one also has a total of 16.

Prove that there are exactly two shading patterns (not counting rotations or reflections) which have a total of 17.

2	1	2
3	2	2
1	2	1

Level: Intermediate Ref No: M123 Puzz Points: 23

[Maclaurin 2010 Q6] Every day for the next eleven days I shall eat exactly one sandwich for lunch, either a ham sandwich or a cheese sandwich. However, during that period I shall never eat a ham sandwich on two consecutive days.
In how many ways can I plan my sandwiches for the next eleven days?

Solution: 233

Level: Intermediate Ref No: M124 Puzz Points: 10

[Cayley 2005 Q1] The edge length, in centimetres, of a solid wooden cube is a whole number greater than two. The outside of the cube is painted blue and the cube is then cut into small cubes whose edge length is 1cm. The number of small cubes with exactly one blue face is ten times the number of small cubes with exactly two blue faces. Find the edge length of the original cube.

Solution: 22cm

Level: Intermediate   Ref No: M129   Puzz Points: 13

[Cayley 2005 Q6] If you have an endless supply of 3×2 rectangular tiles, you can place 100 tiles end to end to tile a 300×2 rectangle. Similarly, you can put k tiles side by side to tile a $3k \times 2$ rectangle.
Find the values of the integers k and m for which it is possible to a $6k \times m$ rectangle with 3×2 tiles.

Solution: Possible to tile any $6k \times m$ rectangle provided $m > 1$ and $k > 0$.

Level: Intermediate   Ref No: M130   Puzz Points: 15

[Hamilton 2005 Q1] Find the number of different ways that 75 can be expressed as the sum of two or more consecutive positive integers. (Writing the same numbers in a different order does not constitute a 'different way'.)

Solution: 5

Level: Intermediate   Ref No: M138   Puzz Points: 20

[Maclaurin 2005 Q3] Two points X and Y lie in a plane. Two straight lines are drawn in the plane, passing through X but not through Y. A further n straight lines are drawn in the plane, passing through Y but not through X. No line is parallel to any other line.

Find, in terms of n, the number of regions into which all $n + 2$ lines divide the plane.

Solution: $4n + 3$